Transient Analysis for Thermal and Moisture Behavior of Building Elements

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Abstract: The simultaneous flow of heat and moisture in building elements has received considerable attention in recent literature. In this paper, a transient hygrothermal problem with coupled temperature and moisture for building elements is investigated. A hybrid numerical method of Laplace transformation and the finite difference is first applied to solve its transient hygrothermal problem, in which the temperature and moisture coupling at the inner and outer surfaces is taken into account in the boundary conditions. The general solutions of the governing equations are first solved in the transform domain, and then the inversion to the real domain is completed by the methods of matrix operation and the Fourier series technique.

Key words: Hybrid method; Coupled temperature and moisture; simultaneous

1.INTRODUCTION

Most building materials like concrete, wood, wall finishes, etc., are porous materials, which can contain moisture. Moisture transfer in porous building elements and furnishings does not only result in condensation but also has a significant influence on indoor humidity and air-conditioning loads, especially latent cooling load[1,2]. Many building materials such as gypsum board and wood, etc., can store moisture. Air conditioning system can rapidly remove the stored moisture in the zone air, but removing the adsorbed moisture in the building elements often accounts for a significant fraction of the overall cooling load, especially upon starting the system after a shutdown period.

material, and assumed a very ideal situation at the boundary conditions. Some of the studies in the literature although considered the energy and mass balance at the boundaries, but only at part of the boundaries instead of all the boundaries[9].

For the purpose of a clear evaluation of moisture load, the absorption and desorption process of building materials have, as a result, become a key issue in building studies, and also a significant constituent of building energy conservation technology. Since experimental investigations on the moisture behavior of building elements are expensive and time-consuming, computational study is becoming increasing important. Therefore, there is an increasing demand for calculation methods to evaluate the moisture behavior of building elements.

In porous materials, moisture trends to migrate to the cooler material side under the influence of temperature and vapor density gradients. The heat transmission process through moist materials is very complex and the calculation of heat transfer with heat conduction theory alone is just an approximation. To determine the contribution of the moist capacitance in the building, an analysis that takes into account simultaneous diffusion of heat and mass in the building elements is required.

Physically there exists a coupling effect between temperature and moisture in most building materials. It has been verified that the influence of the coupling effect becomes significant as the ratio of thermal conductivity to moisture diffusivity approaches unity. So far, there have been many studies[3,8]concerned with the interactive effect between temperature and moisture, experimentally or analytically. However, for the convenience of analysis, most of the analytical investigations only considered the coupling effect between temperature and moisture within the

In the present paper, a transient hygrothermal problem with coupled temperature and moisture for building elements is investigated. A hybrid numerical method is adopted in the study. First, the general solutions of the governing equations are solved in the transform domain by the method of combining the Laplace transformation and the finite difference. The inverse to the real domain is completed by the method of matrix operation and Fourier series technique. Finally, the transient distributions of the temperature and moisture in the wall are obtained.

2.MATHEMATICAL MODEL

A typical heat and mass transfer problem is governed by Luikov's equations[10]. In the present study, one-dimensional governing equations with coupled temperature and moisture for wall are considered, and the effect of the absorption or desorption heat is added. Material properties and pressure are considered to be constant throughout the materials. A local thermodynamic equilibrium between the fluid and the porous matrix is assumed. The coupled governing equations can be expressed as followings[11]

$$\rho C_p \frac{\partial T}{\partial t} = k\frac{\partial^2 T}{\partial x^2} + \rho C_m(\varepsilon h_{lv} + \gamma)\frac{\partial m}{\partial t} \qquad (1)$$

$$\rho C_m \frac{\partial m}{\partial t} = D_m \frac{\partial^2 m}{\partial x^2} + D_m \delta \frac{\partial^2 T}{\partial x^2} \qquad (2)$$

Where T is the temperature, m is the moisture potential, k and D_m are the thermal and moisture conductivity coefficients, respectively, C_p and C_m are the heat and moisture capacities of the material, respectively, ρ is the material density, h_{lv} is the heat of evaporative phase-change, γ represents the heat of absorption or desorption, δ is the thermogradient coefficient, and ε is the ration of the vapor diffusion coefficient to the coefficient of the total moisture diffusion. All the material properties mentioned above are effective properties. The moisture potential m is related to the moisture content C as

$$C = C_m m$$

The coupling diffusion governing Eqs.(1) and Eqs.(2) contain not only general diffusion equations but also some source or sink terms. The governing equation(1) express the balance of thermal energy within the wall; the last term in this equation represents the heat source or heat sinks due to liquid-to-vapor phase-change and to the heat of absorption or desorption. Similarly, Eqs.(2) expresses the balance of moisture within the wall; the last term in this equation depicts the moisture source or sink with respect to the temperature gradient.

To simplify the notation, dividing Eqs.(1) by ρC_p and using the expression for $\frac{\partial^2 T}{\partial x^2}$ obtained from Eqs.(1) into Eqs.(2), then rearranging these two equations to yield

$$L\frac{\partial^2 T}{\partial x^2} = \frac{\partial T}{\partial t} - \nu \frac{\partial m}{\partial t} \qquad (3)$$

$$D\frac{\partial^2 m}{\partial x^2} = \frac{\partial m}{\partial t} - \lambda \frac{\partial T}{\partial t} \qquad (4)$$

where $L = \frac{k}{\rho C_p}$, $D = \frac{kD_m}{\rho C_m[k + D_m\delta(\varepsilon h_{lv} + \gamma)]}$,

$\nu = \frac{C_m(\varepsilon h_{lv} + \gamma)}{C_p}$, $\lambda = \frac{C_p D_m \delta}{C_m[k + D_m\delta(\varepsilon h_{lv} + \gamma)]}$,

ν and λ are positive coupling coefficients due to moisture migration and heat conduction, respectively. L and D are always positive expressing the equivalent temperature diffusion coefficient and the equivalent moisture diffusion coefficient, respectively. The moisture in Eqs.(3) will play the role of a heat source for the temperature distribution, if the moisture rate is positive (i.e., $\frac{\partial m}{\partial t}$ >0), and act as a heat sink if the moisture rate is negative(i.e., $\frac{\partial m}{\partial t}$ <0). Similarly, the temperature may play the role of a moisture source or a moisture sink, depending on the temperature rate being positive or negative.

At the boundaries of the wall, the latent heat of vaporization becomes part of the energy balance, and the mass diffusion caused by the temperature and moisture gradients affects the mass balance. Thus the associated hygrothermal boundary and initial conditions are expressed as followings

$$k\frac{\partial T(x_1,t)}{\partial x} = h_{c1}[T(x_1,t) - T_{\infty 1}] + (1-\varepsilon)h_{lv}h_{m1}[m(x_1,t) - m_{\infty 1}] \quad (5)$$

$$-k\frac{\partial T(x_2,t)}{\partial x} = h_{c2}[T(x_2,t) - T_{\infty 2}] + (1-\varepsilon)h_{lv}h_{m2}[m(x_2,t) - m_{\infty 2}] \quad (6)$$

$$D_m\frac{\partial m(x_1,t)}{\partial x} + D_m\delta\frac{\partial T(x_1,t)}{\partial x} = h_{m1}[m(x_1,t) - m_{\infty 1}] \quad (7)$$

$$-D_m\frac{\partial m(x_2,t)}{\partial x} - D_m\delta\frac{\partial T(x_2,t)}{\partial x} = h_{m2}[m(x_2,t) - m_{\infty 2}] \quad (8)$$

$$T(x,0) = T_0 \quad (9)$$

$$m(x,0) = m_0 \quad (10)$$

3.SOLVING METHOD

Applying the Laplace transformation to Eqs.(3), (4), (5), (6), (7) and (8) with respect to t, they become

$$L\frac{d^2\overline{T}}{dx^2} = S\overline{T} - T_0 - \nu S\overline{m} + \nu m_0 \quad (11)$$

$$D\frac{d^2\overline{m}}{dx^2} = S\overline{m} - m_0 - \lambda S\overline{T} + \lambda T_0 \quad (12)$$

$$k\frac{d\overline{T}(x_1,S)}{dx} = h_{c1}[\overline{T}(x_1,S) - \frac{T_{\infty 1}}{S}] + (1-\varepsilon)h_{lv}h_{m1}[\overline{m}(x_1,S) - \frac{m_{\infty 1}}{S}] \quad (13)$$

$$-k\frac{d\overline{T}(x_2,S)}{dx} = h_{c2}[\overline{T}(x_2,S) - \frac{T_{\infty 2}}{S}] + (1-\varepsilon)h_{lv}h_{m2}[\overline{m}(x_2,S) - \frac{m_{\infty 2}}{S}] \quad (14)$$

$$D_m\frac{d\overline{m}(x_1,S)}{dx} + D_m\delta\frac{d\overline{T}(x_1,S)}{dx} = h_{m1}[\overline{m}(x_1,S) - \frac{m_{\infty 1}}{S}] \quad (15)$$

$$-D_m\frac{d\overline{m}(x_2,S)}{dx} - D_m\delta\frac{d\overline{T}(x_2,S)}{dx} = h_{m2}[\overline{m}(x_2,S) - \frac{m_{\infty 2}}{S}] \quad (16)$$

Where $\overline{T}(S)$ and $\overline{m}(S)$ are the Laplace transformation of $T(t)$ and $m(t)$, respectively; and S is Laplace transformation parameter. Applying central finite difference in Eqs.(11) and (12), we obtain the following discretized equations

$$L\frac{\overline{T}_{j+1} - 2\overline{T}_j + \overline{T}_{j-1}}{\Delta x^2} - S\overline{T}_j + \nu S\overline{m}_j = \nu m_0 - T_0 \quad (17)$$

$$D\frac{\overline{m}_{j+1} - 2\overline{m}_j + \overline{m}_{j-1}}{\Delta x^2} - S\overline{m}_j + \lambda S\overline{T}_j = -m_0 + \lambda T_0 \quad (18)$$

where $\Delta x = \frac{l}{N-1}$, l represent the thickness of the wall and N is the total node number. Substituting boundary conditions of Eqs. (13)---(16) into Eqs. (17) and (18) and writing them in matrix form, we obtain the following equations

$$\{[A] - S[I]\}\{\overline{T}_j\} + \{[B] + \nu S[I]\}\{\overline{m}_j\} = \{C\} \quad (19)$$

$$\{[F] + \lambda S[I]\}\{\overline{T}_j\} + \{[G] - S[I]\}\{\overline{m}_j\} = \{H\} \quad (20)$$

where $[I]$ is a unit matrix and

$$[A] = \begin{bmatrix} -\frac{2L}{\Delta x^2} - \frac{2Lh_{c1}}{k\Delta x} & \frac{2L}{\Delta x^2} & 0 & \cdots & 0 & 0 \\ \frac{L}{\Delta x^2} & -\frac{2L}{\Delta x^2 S} & \frac{L}{\Delta x^2} & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & \cdots & 0 & \frac{L}{\Delta x^2} & -\frac{2L}{\Delta x^2} & \frac{L}{\Delta x^2} \\ 0 & \cdots & 0 & 0 & \frac{2L}{\Delta x^2} & -\frac{2L}{\Delta x^2} - \frac{2Lh_{c2}}{k\Delta x} \end{bmatrix}$$

$$[G]=\begin{bmatrix} -\frac{2D}{\Delta x^2}+\frac{2D\delta(1-\varepsilon)h_{lv}h_{m1}}{k\Delta x}-\frac{2h_{m1}D}{\Delta x D_m} & \frac{2D}{\Delta x^2} & 0 & \cdots & 0 & 0 \\ \frac{D}{\Delta x^2} & -\frac{2D}{\Delta x^2} & \frac{D}{\Delta x^2} & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \cdots & \frac{D}{\Delta x^2} & -\frac{2D}{\Delta x^2} & \frac{D}{\Delta x^2} \\ 0 & 0 & \cdots & 0 & \frac{2D}{\Delta x^2} & -\frac{2D}{\Delta x^2}+\frac{2D\delta(1-\varepsilon)h_{lv}h_{m2}}{k\Delta x}-\frac{2h_{m2}D}{\Delta x D_m} \end{bmatrix}$$

$$[B]=\begin{bmatrix} -\frac{2L(1-\varepsilon)h_{lv}h_{m1}}{k\Delta x} & 0 & \cdots & 0 & 0 \\ 0 & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \cdots & 0 & 0 \\ 0 & 0 & \cdots & 0 & -\frac{2(1-\varepsilon)Lh_{lv}h_{m2}}{k\Delta x} \end{bmatrix}$$

$$[C]=\begin{bmatrix} \nu m_0-T_0-\frac{2Lh_{c1}T_{\infty1}}{k\Delta xS}-\frac{2L(1-\varepsilon)h_{lv}h_{m1}m_{\infty1}}{k\Delta xS} \\ \nu m_0-T_0 \\ \vdots \\ \nu m_0-T_0 \\ \nu m_0-T_0-\frac{2Lh_{c2}T_{\infty2}}{k\Delta xS}-\frac{2L(1-\varepsilon)h_{lv}h_{m2}m_{\infty2}}{k\Delta xS} \end{bmatrix}$$

$$[F]=\begin{bmatrix} \frac{2Dh_{c1}\delta}{k\Delta x} & 0 & \cdots & 0 & 0 \\ 0 & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \cdots & 0 & 0 \\ 0 & 0 & \cdots & 0 & \frac{2Dh_{c2}\delta}{k\Delta x} \end{bmatrix}$$

$$[H]=\begin{bmatrix} -m_0+\lambda T_0+\frac{2D\delta h_{c1}T_{\infty1}}{k\Delta xS}+\frac{2D\delta(1-\varepsilon)h_{lv}h_{m1}m_{\infty1}}{k\Delta xS}-\frac{2h_{m1}m_{\infty1}D}{\Delta xD_mS} \\ -m_0+\lambda T_0 \\ -m_0+\lambda T_0 \\ \vdots \\ -m_0+\lambda T_0+\frac{2D\delta h_{c2}T_{\infty2}}{k\Delta xS}+\frac{2D\delta(1-\varepsilon)h_{lv}h_{m2}m_{\infty2}}{k\Delta xS}-\frac{2h_{m2}m_{\infty2}D}{\Delta xD_mS} \end{bmatrix}$$

Eqs. (19) and (20) can be decoupled by eliminating either $\{\overline{T}_j\}$ or $\{\overline{m}_j\}$ between them. The resulting decoupled equations can be expressed as followings

$$\left[\overline{X}\right]\left\{\overline{T}_j\right\}=\left[\overline{X}^*\right] \tag{21}$$

$$\left[\overline{Y}\right]\left\{\overline{m}_j\right\}=\left[\overline{Y}^*\right] \tag{22}$$

where

$$\left[\overline{X}\right]=\{[F]+\lambda S[I]\}-\{[G]-S[I]\}\{[B]+\nu S[I]\}^{-1}\{[A]-S[I]\}$$

$$\left[\overline{X}^*\right]=\{[H]\}-\{[G]-S[I]\}\{[B]+\nu S[I]\}^{-1}\{[C]\}$$

$$\left[\overline{Y}\right]=\{[G]-S[I]\}-\{[F]+\lambda S[I]\}\{[A]-S[I]\}^{-1}\{[B]+\nu S[I]\}$$

$$\left[\overline{Y}^*\right]=\{[H]\}-\{[F]+\lambda S[I]\}\{[A]-S[I]\}^{-1}\{[C]\}$$

The inverse Laplace transformation of the complicated Eqs.(21) and (22) can be completed by the residue theorem or Fourier series technique. In this paper, the authors adopt the later method.

The numerical results and application will be discussed in detail in another paper.

4.RESULTS AND DISCUSSIONS

In this paper, a model of coupled heat and moisture transfer through building elements is established based on Luikov's equations, energy balance and mass balance. In this model, the effect of absorption or desorption heat is added; energy and mass balance at all the boundaries is considered.

A hybrid numerical method of Laplace transformation and the finite difference is applied to solve the transient hygrothermal problem of building elements. Traditional calculation methods for the coupled governing equations are mainly transfer function method and numerical method. Transfer function method is more efficient for solving long-time hygrothermal problems than numerical method because of its shorter time steps. But transfer function method assumed the material properties are constant with time, this may cause the solution result deviate from the normal value. Numerical method can avoid the problem mentioned above. But in order to avoid numerical oscillations or instability, the time step is need to be small. The hybrid numerical method integrate the advantages of transfer function method and numerical method. It can provide more precise results than transfer function method and numerical method.

5.CONCLUSIONS

In this paper, based on Luikov's equations, energy balance and mass balance, a model of coupled heat and moisture transfer through building elements is established. A hybrid numerical method of Laplace transformation and the finite difference is firstly applied to solve the transient hygrothermal problem of building elements. It provide more precise results than traditional methods.

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